Math Tips for Students

GRADES 6 AND UP
Essential Facts & Concepts

MATHNASIUM®
The Math Learning Center

INTRODUCTION

This booklet contains math facts and concepts that should be well-known by every student in 6th grade and up. They form the fabric of knowledge — the foundation necessary for success in the middle school and high school math classroom. Most teachers assume that their students know these.

When these facts and concepts are at your fingertips, new material encountered becomes an extension of things you already know. Without them, it seems like every new topic has to be learned from the beginning.

As you ask yourself questions based on these facts and concepts, you will be pleased by how many of them you already know. At the same time, you might be surprised by some of the things you don't know (yet).

We encourage you to study and internalize the following.

WHOLES AND PARTS

• The whole equals the sum of its parts.
• Each part equals the whole minus the sum of the other parts.

ZERO — 0

• Zero counts "the number of" when the answer is "none."
• Zero is an even number.
• Zero is the only number that is neither positive nor negative.
• Any number plus its opposite is zero. [3 + ⁻3 = 0]
• Any number plus zero is equal to that number. [369 + 0 = 369]
• Any number minus zero is equal to that number. [24 – 0 = 24]
• Every number is a factor of zero. That is, every number "goes into" zero exactly no (0) times, with "nothing left over."
• Division by zero is not possible.

ONE — 1

- One (1) times any number is equal to that number. [667 x 1 = 667]
- Any number divided by one (1) is equal to that number. [42 ÷ 1 = 42]
- One (1) is a factor of every number. [6 = 1 x 6]
- Negative one (‾1) is a factor of every number. [6 = ‾1 x ‾6]
- Every number times its reciprocal equals one (1). [$\frac{2}{3}$ x $\frac{3}{2}$ = 1]
- A proper fraction has a value less than one whole (1).
- An improper fraction has a value greater than one whole (1).
- Unity is the fraction that has a value equal to one whole (1).

COMMON FRACTION — DECIMAL EQUIVALENCES

$\frac{1}{2}$ = 0.5

$\frac{1}{3}$ = 0.333 … = 0.33$\frac{1}{3}$

$\frac{2}{3}$ = 0.666 … = 0.66$\frac{2}{3}$

$\frac{1}{4}$ = 0.25

$\frac{3}{4}$ = 0.75

$\frac{1}{5}$ = 0.2

$\frac{1}{6}$ = 0.1666 … = 0.16$\frac{2}{3}$

$\frac{5}{6}$ = 0.8333 … = 0.83$\frac{1}{3}$

$\frac{1}{7}$ = 0.142857 … = 0.14$\frac{2}{7}$

$\frac{2}{7}$ = 0.285714 … = 0.28$\frac{4}{7}$

$\frac{1}{8}$ = 0.125 … = 0.12$\frac{1}{2}$

$\frac{3}{8}$ = 0.375 … = 0.37$\frac{1}{2}$

$\frac{5}{8}$ = 0.625 = 0.62$\frac{1}{2}$

$\frac{7}{8}$ = 0.875 = 0.87$\frac{1}{2}$

$\frac{1}{9}$ = 0.111 = 0.11$\frac{1}{9}$

$\frac{1}{10}$ = 0.1

$\frac{1}{16}$ = 0.0625 = 0.06$\frac{1}{4}$

$\frac{1}{20}$ = 0.05

$\frac{1}{25}$ = 0.04

$\frac{1}{40}$ = 0.025 = 0.02$\frac{1}{2}$

$\frac{1}{50}$ = 0.02

$\frac{1}{100}$ = 0.01

$\frac{1}{1,000}$ = 0.001

$\frac{1}{1,000,000}$ = 0.000001

USEFUL SQUARES

12^2 = 144 = one gross, which is a dozen dozens.

13^2 = 169	16^2 = 256	30^2 = 900
14^2 = 196	20^2 = 400	40^2 = 1,600
15^2 = 225	25^2 = 625	50^2 = 2,500

MULTIPLYING TO 100

Factors "go into" a number evenly. [1, 2, 3, 4, 6, and 12 are all the factors of 12.]

$1 \times 100 = 100$	$5 \times 20 = 100$	$9 \times 11\frac{1}{9} = 100$	$20 \times 5 = 100$
$2 \times 50 = 100$	$6 \times 16\frac{2}{3} = 100$	$10 \times 10 = 100$	$25 \times 4 = 100$
$3 \times 33\frac{1}{3} = 100$	$7 \times 14\frac{2}{7} = 100$	$11 \times 9\frac{1}{11} = 100$	$50 \times 2 = 100$
$4 \times 25 = 100$	$8 \times 12\frac{1}{2} = 100$	$12 \times 8\frac{1}{3} = 100$	$100 \times 1 = 100$

FRACTIONS

- A half plus a half equals a whole.
  $$\frac{1}{2} + \frac{1}{2} = 1$$
- One-fourth plus one-half equals three fourths.
 $$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$$
- Three-fourths plus three-fourths equals one and one half.
  $$\frac{3}{4} + \frac{3}{4} = 1\frac{1}{2}$$
- Half-of-a-half is a quarter ($\frac{1}{4}$).
- Half-of-a-quarter is an eighth ($\frac{1}{8}$).
- Half-of-an-eighth is a sixteenth ($\frac{1}{16}$).
- Half-of-a-sixteenth is a thirty-second ($\frac{1}{32}$).
- You can cut a line segment in half forever.

- A dime is a tenth ($\frac{1}{10}$) of a dollar because 10 dimes make a dollar.
- A penny is a hundredth ($\frac{1}{100}$) of a dollar because 100 pennies make a dollar.
- A quarter is a fourth ($\frac{1}{4}$) of a dollar because 4 quarters make a dollar.
- A nickel is a twentieth ($\frac{1}{20}$) of a dollar because 20 nickels make a dollar.

- A quart is a quarter of a gallon — four quarts make a gallon.
- A pint is half of a quart — two pints make a quart.
- A cup is half of a pint — two cups make a pint.
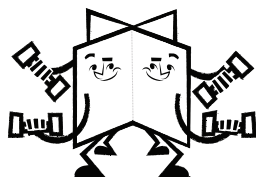
USEFUL MULTIPLES OF 15 AND 16

A number "goes into" its multiples. [0, 6, 12, and 18 are the first four multiples of 6.]

2 x 15 = 30	2 x 16 = 32
3 x 15 = 45	3 x 16 = 48
4 x 15 = 60	4 x 16 = 64
5 x 15 = 75	5 x 16 = 80
6 x 15 = 90	6 x 16 = 96

PERCENTS

"Percent" means *for each hundred.*

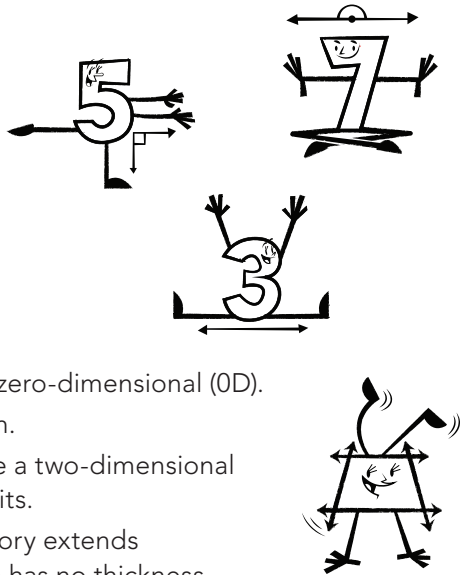|	
0%	means "none of it."
10%	means "a tenth ($\frac{1}{10}$) of it."
25%	means "a quarter ($\frac{1}{4}$) of it."
$33\frac{1}{3}$%	means "a third ($\frac{1}{3}$) of it."
50%	means "half ($\frac{1}{2}$) of it."
$66\frac{2}{3}$%	means "two-thirds ($\frac{2}{3}$) of it."
75%	means "three-quarters ($\frac{3}{4}$) of it."
100%	means "all of it."
200%	means "all of it, twice."
$\frac{1}{2}$%	is less than 1%, and is not the same as 50%.

PRIMES

- A prime number has exactly two distinct factors: 1 and itself.
- 0, 1, 51, 57, 87, and 91 are not prime numbers.
- Two (2) is the only even prime number.
- Numbers are relatively prime if they have no common factors other than one (1), that is, their Greatest Common Factor (GCF) is 1.
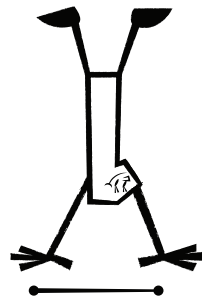
GEOMETRY

- A right angle contains 90°.
- A straight angle contains 180°.
- A full circle contains 360°.
- Two adjacent right angles make a straight line.
- The three dimensions are called:
    length, *width*, and *depth*, or
    length, *width*, and *height*.
- A **point** occupies no space and is zero-dimensional (0D).
- A **line** has length (1D) and no width.
- **Area** is the amount of space inside a two-dimensional figure (2D), measured in square units.
- A **plane** is a flat surface that in theory extends infinitely in two directions. A plane has no thickness.
- **Volume** is the amount of space inside a three-dimensional figure (3D), measured in cubic units.
- **Space** is the infinite extension of the three dimensions of length, width, and depth.
- **Time** is often considered to be the fourth dimension.
- **Parallel lines** are always the same distance apart. Parallel lines do not intersect.
- **Perpendicular lines** meet to form four right angles.
- **Distance** is the amount of space between two points, measured in linear units, along a given path. Distance is one-dimensional (1D).
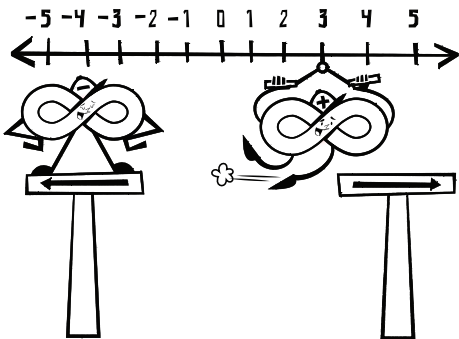
- Distance is always a *positive* quality.
- The **absolute value** of a number is its distance from zero (0).
- Because the absolute value is a distance, it is always a positive number or zero, and is *never* negative.
- The *sum of the angles* of a triangle is 180°.

PREFIXES & SUFFIXES

poly- means "many"

uni- and mono- mean 1

bi- means 2

tri- means 3

quad- means 4

quin- and pent- mean 5

hex- and *sex-* mean 6

sept- and *hept-* mean 7

oct- means 8

non- means 9

nano- means billionth ($\frac{1}{1,000,000,000}$)

micro- means millionth ($\frac{1}{1,000,000}$)

milli- means thousandth ($\frac{1}{1,000}$)

centi- means hundredth ($\frac{1}{100}$)

deci- means tenth ($\frac{1}{10}$)

dec- means 10

cent- means hundred (100)

kilo- means thousand (1,000)

mega- means million (1,000,000)

mil- means thousand (1,000)

dia- means "across"

-meter means "measure" (diameter = the "measure across" the circle)

-gon means "angle" (polygon = many angles)

-lateral means "side" (quadrilateral = four sides)

OTHER DEFINITIONS

Mile means "1,000 paces."

Million means "1,000 thousands."

A *millennium* is 1,000 years.

Squared means "to the second power."

Cubed means "to the third power."

Denominate means "to name."

Numerate means "to specify the number of … "

The **denominator** of a fraction is "the name of the fraction," and tells "the number of equal parts in the whole."

Since the **denominator** gives the "name" of the fraction, the denominator is a *noun*.

The **numerator** of a fraction is "the number of equal parts being used."

Since the **numerator** tells us "the number of," the numerator is an *adjective*.
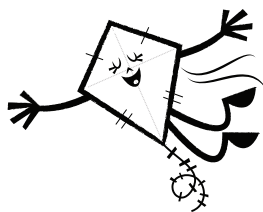
IMPORTANT CONCEPTS

- Every *thing* has a **quantity** (the *number* of) and a **denomination** (a *name*).

- **The Law of SAMEness**: You can only add and subtract things that have the same name (a common denomination).

- The **decimal point** (.) separates the wholes (on the left side of the decimal) from the parts (on the right side of the decimal).

- The **fraction bar** (–) separates the numerator (on the top of the fraction) from the denominator (on the bottom of the fraction). The proper name for the fraction bar is "vinculum."

- The **fraction bar** also means:
    - the numerator "out of" the denominator,
    - the numerator "per" the denominator,
    - the numerator "divided by" the denominator, and the denominator "divided into" the numerator.

- A **difference** is a comparison of two numbers by subtraction.
    - Comparing 6 and 8 by **differences** gives "6 is 2 less than 8."

- A **ratio** is a comparison of two numbers by *division*.
    - Comparing 6 and 8 by **ratio** gives "$\frac{6}{8}$, which means 6 is three-fourths ($\frac{3}{4}$) of 8."

- A **proportion** is an equation that says that two ratios are equal.

    $\frac{6}{9} = \frac{8}{12}$ is a **proportion**.

    - This says that 6 and 9 have the same ratio as 8 and 12, since both equal two-thirds ($\frac{2}{3}$) [6 is two-thirds ($\frac{2}{3}$) of 9, and 8 is two-thirds ($\frac{2}{3}$) of 12].
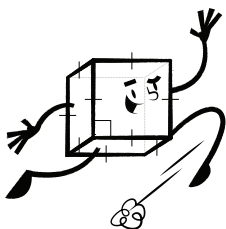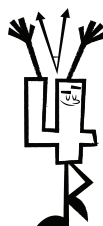
- The phrase "**the quantity**" has two meanings:

    1) the "**amount**" — "the quantity of sand used is 6 pounds"

    2) the "**one composed of the many**"

    "$(2x + 3y)^2$" is pronounced,

    "$2x$ plus $3y$, the quantity squared."

    "$2x + 3y^2$" is pronounced,

    "$2x$ plus $3y$ squared."

    "$2x + (3y)^2$" is pronounced,

    "$2x$ plus the quantity $3y$ squared."

- When multiplying two binomials, learn to "**FOIL**" (**F**irst, **O**utside, **I**nside, **L**ast) mentally, that is, to combine the "**OI**" mentally and write down just the results.

    $(x + 3)(x + 4) = x^2 + 7x + 12$ [mentally combine $4x + 3x$]

    rather than writing down

    $(x + 3)(x + 4) = x^2 + 4x + 3x + 12 = x^2 + 7x + 12.$

    ○ Learning to do this will save time and will make learning advanced concepts in algebra II and precalculus much easier.

- Be able to apply the **Pythagorean Theorem** *forward* and *backward* in various situations.
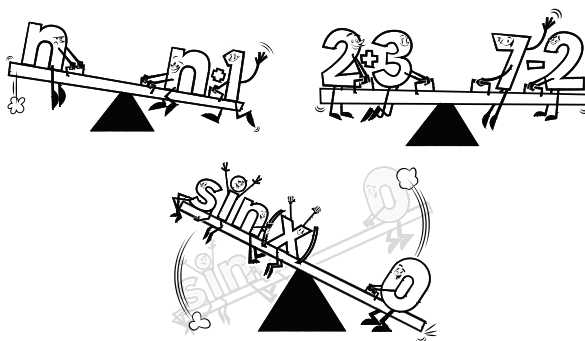
    ○ In a right triangle —

    Forward: $a^2 + b^2 = c^2$, and
    Backward: $a^2 = c^2 - b^2$, and $b^2 = c^2 - a^2$,

    ○ Where a and b are legs of the triangle and c is the hypotenuse.

    This is the most often-used formula in math and science.

UNITS OF MEASURE

1 foot	=	12 inches
1 yard	=	3 feet = 36 inches
1 mile	=	5,280 feet = 1,760 yards
1 minute	=	60 seconds
1 hour	=	60 minutes
1 day	=	24 hours
1 week	=	7 days
1 fortnight	=	2 weeks = 14 days
1 year	=	365 days (regular year),
	=	366 days (leap year)
1 year	=	12 months
1 year	=	52 weeks and 1 day (regular year)
	=	52 weeks and 2 days (leap year)
1 score	=	20 things
1 score (of years)	=	20 years = 2 decades
1 decade	=	10 years = $\frac{1}{2}$ score
1 century	=	100 years = 10 decades = 5 scores
1 millennium	=	1,000 years = 100 decades = 10 centuries
1 pound	=	16 ounces
1 ton	=	2,000 pounds
1 gallon	=	4 quarts
1 quart	=	2 pints
1 pint	=	2 cups
1 cup	=	8 fluid ounces
1 gross	=	a dozen dozens (144 things)

1 fathom	=	6 feet = 2 yards
1 league	=	3 miles
1 furlong	=	$\frac{1}{8}$ of a-mile
speed of light	$\approx$	186,000 miles per second
	$\approx$	300,000 kilometers per second
1 light-year	$\approx$	6,000,000,000,000 miles
1 parsec	$\approx$	3.3 light-years

1 inch equals *exactly* 2.54 centimeters.

A kilometer is a little more than half a mile.

A liter is a little more than a quart.

A kilogram = 1,000 grams and equals about 2.2 pounds.

NUMBER SYSTEMS

Natural Numbers ($\mathbb{N}$)
(sometimes called **Counting Numbers**)

The natural numbers are the positive numbers that do not contain fractions.

EXAMPLES: $\mathbb{N} = \{1, 2, 3 \dots\}$

Whole Numbers ($\mathbb{W}$)

The whole numbers are 0 and the positive numbers that do not contain fractions or decimals (the natural numbers and 0).

EXAMPLES: $\mathbb{W} = \{0, 1, 2, 3 \dots\}$

Integers ($\mathbb{Z}$)

The integers are the *positive* and *negative* whole numbers, including 0 (the whole numbers and their *opposites*).

EXAMPLES: $\mathbb{Z} = \{\dots {}^-3, {}^-2, {}^-1, 0, 1, 2, 3 \dots\}$

Rational Numbers ($\mathbb{Q}$)

The rational numbers have the form $\frac{a}{b}$, where a and b are whole numbers and $b \neq 0$. That is, they are all the numbers that can be written as **fractions** (as **ratios**).

All rational numbers can be written as common fractions, as decimal fractions, and as percents. The decimal form of a rational number is either a *repeating* decimal or a *terminating* decimal.

EXAMPLES: $\frac{1}{4} = 0.25 = 25\%$ $\qquad \frac{1}{3} = 0.\overline{3} = 33\frac{1}{3}\%$

Irrational Numbers ($\mathbb{I}$)

The irrational numbers are numbers that cannot be written as fractions. Their decimal representations do not repeat and do not terminate.

EXAMPLES: $\pi = 3.14159\dots$ $\sqrt{2} = 1.414213\dots$
$x = 0.010010001\dots$

Complex Numbers ($\mathbb{C}$)

The complex numbers are all the real numbers and the numbers that involve the square root of negative one ($\sqrt{{}^-1}$). Complex numbers have the general form $a + bi$, where a and b are real numbers and $i = \sqrt{{}^-1}$. a is the *real* part and bi is called the *imaginary* part.

(For real numbers, $b = 0$. For **pure** *imaginary* numbers, $a = 0$.)

EXAMPLES: $3 + 2i$, $5 - 6i$, $4i$ ($a = 0$), 11 ($b = 0$).

A DIFFERENT WAY TO THINK ABOUT PERCENT

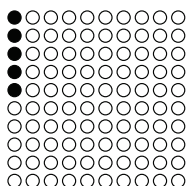"**Per**" means "**for each**" and **cent** means "**100**." So **percent** means "**for each 100**."

EXAMPLE: Find **5**% of **300**.

5% means "5 for each 100." **So count ...**
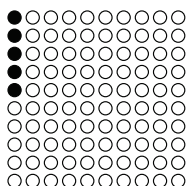
5 for the *first* 100,   **5** for the *second* 100,   and **5** for the *third* 100.
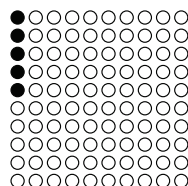
| 5 | + | 5 | + | 5 | = 15 |

So, **5% of 300 is 15**.

Try these:

1) 7% of 300 = _____

2) 6% of 500 = _____

3) 15% of 300 = _____

4) 25% of 400 = _____

5) 20% of 500 = _____

6) 12% of 300 = _____

7) 6% of 200 = _____

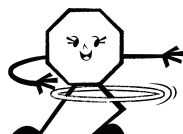8) $6\frac{1}{2}$% of 200 = _____

9) 8% of 50   = _____

10) 7% of 50   = _____
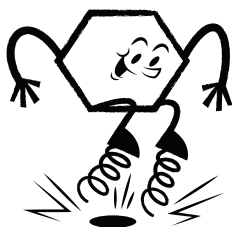
11) 6% of 150 = _____

12) 12% of 250 = _____

13) 8% of 225 = _____

14) 7% of 250 = _____

CONCLUSION

These tips will help you develop a true understanding — and love — of math.

Practice these techniques and tricks and you will feel more comfortable doing math than you ever thought possible.

If you master these concepts, the new material you encounter in high school and college will feel like a logical extension of what you already know.

MATHNASIUM'S PHILOSOPHY:

Children don't hate math. They hate being confused and intimidated by math. With understanding comes passion. And with passion comes growth — a treasure is unlocked.
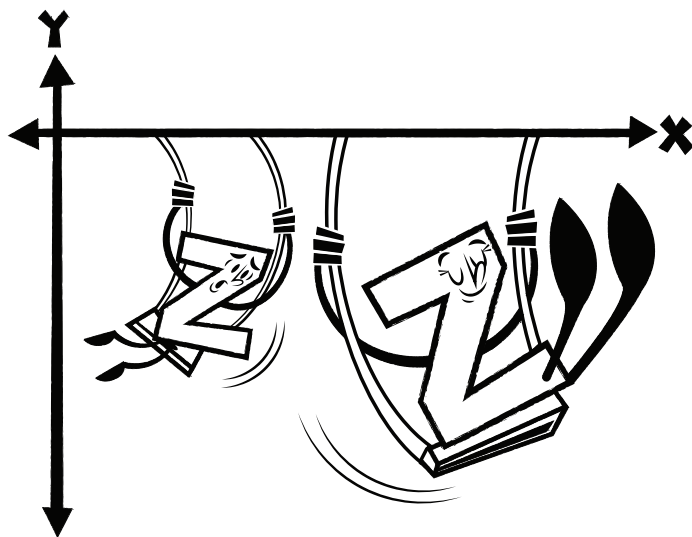
HOW MATHNASIUM BEGAN

In the 1990s, education industry pioneers **Peter Markovitz** and **David Ullendorff** recognized that, to truly be successful in school and in life, students need a solid understanding of mathematics. There was a vast disconnect, though, between students' learning skills and the math curriculum they were taught in school.

To address that gap, they founded Mathnasium — a math-only learning center committed to providing the world's best instruction. Their goal: teach children how to think, with the skills to succeed in math and the confidence to achieve their academic potential.

After an exhaustive search, they found the perfect person to help them achieve their vision: **Larry Martinek**, a beloved educator, teacher trainer and curriculum consultant. Larry and Mathnasium's expert team spent years refining the most powerful teaching methods and materials into the comprehensive, industry-leading **Mathnasium Method**™.

The trio opened the first Mathnasium Learning Center in Los Angeles in late 2002 and began franchising in 2003. The extraordinary dedication of Mathnasium's local center owners and staff, combined with Mathnasium's child-centered approach, proved wildly successful. Mathnasium has become an award-winning, world-class math education company, and global demand for our service has soared.

Today, Mathnasium changes the lives of more than 100,000 children in neighborhood learning centers on five continents.

Mathnasium is your neighborhood math-learning center. Whether your child struggles with fundamental concepts or wants to challenge themselves through enrichment opportunities, we can transform their understanding of math. We do it in a positive environment that children love. And we give them critical thinking and problem-solving skills they need to succeed in class and in life.

The Mathnasium Method™

COMPREHENSIVE ASSESSMENTS

- Oral and written diagnostics pinpoint exact strengths and weaknesses
- Evaluations are conducted regularly to confirm skills retention and guide learning plan updates
- Results are shared with parents upon joining and in progress reports

FULLY CUSTOMIZED LEARNING PLAN

- Approach builds upon student's knowledge and fills in foundational gaps
- Mental, verbal, visual, tactile, and written teaching methods adapt to any learning style
- Time-tested, proprietary materials and instruction ensure individual mastery

EXPERTS WITH A PASSION FOR TEACHING CHILDREN

- Provide face-to-face instruction
- Teach at the perfect pace for each child
- Build confidence through care and encouragement
- Use Socratic questioning to foster independent learning and problem-solving